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Vertex splines, finite elements, multivariate splines, interpolation, quasi-interpolation, approximation order, system theory, digital signal processing, rational approximation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The notion of vertex splines in introduced to generalize the univariate spline theory with arbitrary knot sequence to higher dimensions. These are in fact Hermite elements and to facilitate the construction process, the notion of super splines is also introduced. The advantages include efficiency in computing a locally supported basis, guaranteeing the full order of approximation, and various applications to finite element methods, computer-aided geometric design, data analysis, etc. In general, computational schemes are

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) studied and constructed, and interpolation as well as quasi-interpolation problems are solved. Shape-preserved approximation and interpolation by bivariate splines is also studied. Development of general multivariate spline theory including the dimension and basis problems is also a portion of this project. On the other hand, applications to engineering problems are included in our study. Keywords: digital signal processing. (KR)

Multivariate Regression with Emphasis on
Multivariate Spline Methods

FINAL REPORT

by

CHARLES K. CHUI

July 12, 1988

Contract Number DAAG 29-84-K-0154

Texas A&M University
College Station, Texas 77843

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Pages

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II. Participating scientific personnel

Faculty

Charles K. Chui

Graduate students and research associates

G. Chen (Ph.D. Summer, 1987)
H. Diamond
T. X. He
M. J. Lai (Ph.D. Spring, 1989)

III. Brief Outline of Research Findings

During the period supported by the U.S. Army Research Office under Contract Number DAAG 29-84-0154 (Aug. 16, 1987 - Aug. 15, 1988 and one-year extension at no cost), we have obtained many results in the area of *multivariate regression with emphasis on multivariate splines*. Several of these findings are fundamental in the sense that a new area in *Approximation Theory*, called *Multivariate Splines*, has been developed. This area is believed to have a great potential in many important applications to scientific and engineering research, from mathematical modeling to analysis of multidimensional data. The Principal Investigator was selected by CBMS (Conference Board of Mathematical Sciences) through a grant from the National Science Foundation to deliver a series of ten one-hour lectures on this subject in August, 1987. The monograph based on these lectures will appear under the title: *Multivariate Splines*, published by SIAM (Society for Industrial and Applied Mathematics) in the Fall of 1988. Our results have been published or will be published in the open literature as listed on pages 1 - 2. In order to avoid redundancy, this list will also be used as references to this outline of research findings. In addition, since many of these results have been reported in our semi-annual progress reports, we will be brief in the areas that have been reported previously. This project is a continuation of the work performed under ARO contracts No. DAHCO 4-75-G0186, DAAG 29-78-G0097, and DAAG 29-81-K-0133. Hence, this final report may be considered as a follow-up of the final report prepared on Aug. 2, 1984. In particular, the list of publications in Section I includes the follow-up work from the previous contracts. We divide our results in five categories as follows:

1. **Vertex splines and applications.** The results in this area can be found in the articles 1985 [1], 1987 [10], and 1988 [12,15,16] listed in Section I. It is well known that every spline function in one variable can be represented as a B-spline series (cf. de Boor [S1] and Schumaker [S22]). However, only the very special setting where the knot sequence is equally space has a chance for generalization to higher dimensions. There was an attempt in using simplicial splines by Micchelli [S17] and Dahmen and Micchelli [S16], but the computational complexity is somewhat prohibitive and what is worse is that the grid partition is far too complicated. Of course, box splines developed by de Boor and DeVore [S2] and de Boor and Höllig [S3] are easy to compute, but they can only be used for regular grid partition which reduces to the equally spaced-knot setting in the one-variable case (cf. Chui [8] and [10], and Chui, Schumaker, and Utreras [14] for more details). Our initiation in a practical generalization of the arbitrary knot setting to the higher dimensional cases results in the development of what is now called vertex splines (see 1985 [1], 1987 [10], and 1988 [15, 16] in Section I). These locally supported functions in fact reduce to the Hermite splines (cf. [S19]) in the one-variable setting. Vertex splines have several important features: the supports are smallest possible, the approximation order is full, they form a Hermite basis, and they are representable in terms of Bezier nets (cf. Chui [S10] for some examples). The only disadvantage is that the degree is usually too high. Recently, de Boor and Höllig [S4] proved that in the two-variable setting the lowest degree d for C^r spline functions to retain the full order of approximation is $d = 3r + 2$. In 1986 [16], we have indeed proved and constructed vertex splines in this space to give the full order of approximation. In fact, a subspace of *super splines* (to be discussed in the second category) whose basis consists of

vertex splines is sufficient to do the job. To further reduce the degree, we subdivide the triangles to obtain what is called *generalized* vertex splines in 1988 [12] (see Chui [S10] for some examples). For low degrees, vertex splines are indeed generalizations of finite elements, and hence may be used to improve the methods in finite elements when smooth elements are required. Vertex splines also have applications to data analysis (1988 [15]) and computer-aided geometric design (1988 [13]).

2. **General multivariate spline theory and computations.** The results in this area can be found in the articles 1986 [5], 1987 [1], and 1988 [5,9,10] listed in Section I. Box splines are simple generalizations of univariate B-splines with equally spaced knots (see Chui [S10] for direct generalization of Schoenberg's theory [S18]). Hence, the computation of box splines should be quite simple. In fact, recurrence relations for box splines exist (see de Boor and Höllig [S3]) and can be applied to compute box splines efficiently. The computational procedures derived from recurrence formulas in [S3] give the function value at each point, but the same algorithm must be applied again to give the value at another point. In our work (1987 [1]), we gave a computational procedure to obtain the Bézier net of each polynomial piece of a box spline recursively, and hence all values of a box spline are obtained simultaneously. It is well known, however, that box splines may not have minimal supports. The study of minimal supported bivariate splines on the three- and four-directional meshes was initiated by de Boor and Höllig [S5]. In our work (1986 [5]), we discovered that minimally supported splines are sometimes not sufficient to generate all locally supported spline functions by linear combinations of translates and introduced the notion of quasi-minimally supported splines. These together with the minimally supported ones are the "unique" ones with smallest possible supports that can generate all locally supported splines. Their algebraic, analytic, and approximation properties are studied in 1988 [5] and a computational procedure is given in 1988 [9]. Another important problem in multivariate spline theory is to give the exact dimension of the spline space. This problem was first posed by Strang [S23]. Unfortunately, the general problem is still open, although lower and upper bounds were given by Schumaker in [S20] and [S21] respectively. For special grid partitions such as crosscut partitions, and more generally a partition consisting of crosscuts and rays, the exact dimension was given by Chui and Wang [S15]. In construction of vertex splines, it is more convenient to assume existence of higher order derivatives at the vertices. Surprising, the order of approximation is preserved by adding this extra condition. Hence, the notion of super splines was introduced in our paper 1987 [10]. In our paper 1988 [10], we have determined the dimension of any super spline subspace of a spline space determined by crosscuts and rays and have also given sharp upper and lower bounds. These results even reduce to the results in [S15], [S20], and [S21] when the super condition is dropped.

3. **Interpolation by Multivariate Splines.** Our results in this area are in 1986 [4], 1987 [2,4,7,8], and 1988 [8,11,12,13,14] listed in Section I. Obviously this is a very important problem both in theory and applications. We have considered interpolation of both gridded and scattered data and have even considered interpolation with shape-constraints such as positivity, monotonicity, and convexity. For gridded data, the Neumann series approach introduced in 1987 [5] is used in 1988 [8]. This is a fundamental paper in the sense that a new area of interpolation problems is developed. This area may be called "interpolation

with a singular function." More precisely, the Fourier series (or symbol) of a locally supported function, such as a box spline, may have zeros, so that the cardinal interpolation problem studied in de Boor, Höllig, and Riemenschneider [S6] does not have a unique solution. However, it doesn't mean that interpolation is impossible. In fact, we were interested in obtaining an interpolant that gives the full order of approximation. In certain cases, even though the symbol vanishes, the Neumann series might still converge to an interpolant that gives full order of approximation. For a divergent Neumann series, we studied the adjustment of the series so that a required interpolant is obtained. A more general problem on interpolation with a singular function is recently studied by de Boor, Höllig, and Riemennschneider [S7]. For scattered data it is best to use vertex splines. Shape-preserving interpolation was studied in 1988 [11], [13], and [14], and an efficient scheme was obtained in 1987 [7].

A more fundamental question is the poisedness of the interpolation problem. This means that if the number of sample points is the same as the dimension of the space, the problem is to study the existence and uniqueness of the interpolant. For the polynomial case, configurations to place sample points are given in our paper (1987 [4]). This is the first result for the higher dimensional settings. The first paper in the literature that studies the poisedness of bivariate spline interpolation is our paper 1986 [4] where bivariate linear splines are considered. In our paper 1987 [8] we continued our study of interpolation by C^1 quadratic splines on the four-directional mesh. Interpolation by splines in more than two variables has not been studied at this writing. In general, much research is required to yield more satisfactory results.

4. **Quasi-interpolation**. Our results in this area are contained in our papers 1987 [2,3,5] and 1988 [3,6,7,11,14,15,16] listed in Section I. In general, as we pointed out above, the interpolation problem is still open. In any case, a spline interpolant to multi-dimensional data is expansive to obtain. For all practical purposes, however, it is sufficient to give a good approximation of the data but at the same time guarantee the full order of approximation everywhere else. A linear scheme that achieves such an approximant is called a quasi-interpolant. To study this problem, one must first know the order of approximation provided by the spline space. If the space is given by the translates of a single function, then the order of approximation is determined by the zero property of its Fourier transform. This result can be found in Strang and Fix [S24]. In our paper (1987 [2]), the notion of the commutator of this generating function was introduced. The order of approximation is shown to be given by the commutator order. As a result, a multivariate analogue of Marsden's identity is obtained in 1987 [3] which gives a qausi-interpolation scheme. In another attempt to construct other schemes, we introduced the Neumann series approach in 1987 [5] and discovered that partial sums of the (possibly divergent) Neumann series yield a sequence of quasi-interpolation schemes. As pointed out above, this approach lead to the discovery of a new area of research in cardinal interpolation. It should be remarked, however, that only regularly gridded data can be handled with the above mentioned schemes. Along the same line, we studied the necessary grid refinement that allows our quasi-interpolants to preserve certain shapes of the data. This was done in 1988 [11] and 1988 [14]. Our study of quasi-interpolation of scattered data is contained in 1988 [13] where Hermite data information is considered. When only partial information

of the Hermite data is given such as in the study of wind field over a complex terrain (cf. Chui [S9]), the method introduced in 1988 [15] can be used, but further research is required to guarantee the optimal order. To incorporate the quasi-interpolation schemes with other multivariate function classes, a general class of linear operators is considered in 1988 [3]. Here, both integral and summation are used simultaneously to achieve a more general scheme. Asymptotic results are obtained in 1988 [6].

5. **Applications to Engineering Problems**. Our results in this area are reported in 1986 [1,2,3], 1987 [6,9], and 1988 [1,2,4] listed in Section I. One common class of functions in physical sciences and engineering is the collection of rational functions with restricted poles. In electromagnetism, the poles represent the location of point charges, and in mechanics they represent point masses. In system theory, if the poles are restricted to the left half plane, the system (or plant) is stable. In digital signal processing, stability is characterized by keeping all the poles inside the unit circle. In our work 1986 [2], 1987 [6,9], and 1988 [1,2,4], we studied problems in rational approximation for these purposes and introduced the problem of weight characterization. It was our original plan to extend some of these results to the multivariate setting but time has already run out on this project. In real-time applications, one has to go to the state-space description where the system matrix dominates the role of the rational function (see Chui and Chen [S11] and [S13] for details). Kalman decomposition is used to separate the system into four subsystems. However, as pointed out in our paper 1986 [3], the existing proofs are not mathematically rigorous and still require additional work. A standard approach in real-time problems is to use Kalman filtering algorithms. Unfortunately, a matrix Riccati equation must be solved at each time instant to compute the Kalman gain matrix. For efficiency purposes, the steady-state approach is usually used (cf. Chui and Chen [S12]), and in our paper 1986 [1], colored noise processes are considered in this setting. In addition, if the variance matrices of the noise processes are unknown, they have to be estimated in real-time. This problem was studied in 1987 [6].

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The list of manuscripts submitted or published under ARO sponsorship listed in Section I on pages 1 - 2 is used for references. In addition, the following list of monographs, conference proceedings, and papers are also referred to in this report.

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